Faster Binary Curve Software: A Case Study

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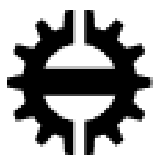

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Elliptic curves over binary fields

Fix m and consider all of the (x, y) solutions over F_{2^m} to the following equation:

$$E : y^2 + xy = x^3 + a_2x^2 + a_6$$

Standardized curves

```
$ openssl ecparam -list_curves
...
  sect163k1 : NIST/SECG/WTLS curve over a 163 bit binary field
  sect163r2 : NIST/SECG curve over a 163 bit binary field
  sect233k1 : NIST/SECG/WTLS curve over a 233 bit binary field
  sect233r1 : NIST/SECG/WTLS curve over a 233 bit binary field
  sect239k1 : SECG curve over a 239 bit binary field
  sect283k1 : NIST/SECG curve over a 283 bit binary field
  sect283r1 : NIST/SECG curve over a 283 bit binary field
  sect409k1 : NIST/SECG curve over a 409 bit binary field
  sect409r1 : NIST/SECG curve over a 409 bit binary field
  sect571k1 : NIST/SECG curve over a 571 bit binary field
  sect571r1 : NIST/SECG curve over a 571 bit binary field
...
```

Carryless multiplication

projects / openssl.git / commit

summary | shortlog | log | commit | commitdiff | tree
(parent: b5c6aab) | patch

x86_64 assembler pack: add x86_64-gf2m module.

author	Andy Polyakov <appro@openssl.org>	
	Mon, 16 May 2011 20:46:45 +0300 (17:46 +0000)	
committer	Andy Polyakov <appro@openssl.org>	
	Mon, 16 May 2011 20:46:45 +0300 (17:46 +0000)	
commit	afebe623c52d2067d1c34d4461ee92924371621d	
tree	96e677f78ce827b6af6f288c6025919e60e0623e	tree | snapshot
parent	b5c6aab57e71015c00d5149f1cf6d03dab1d67ec	commit | diff

x86_64 assembler pack: add x86_64-gf2m module.

projects / openssl.git / commit

summary | shortlog | log | commit | commitdiff | tree
(parent: 3514154) | patch

Add ARMv8 Montgomery multiplication module.

author	Andy Polyakov <appro@openssl.org>	
	Sat, 21 Mar 2015 15:54:17 +0300 (13:54 +0100)	
committer	Andy Polyakov <appro@openssl.org>	
	Mon, 20 Apr 2015 15:39:34 +0300 (14:39 +0200)	
commit	cb2ed545828065e4099137ba3e3568328348f473	
tree	ac7ef190bdfc1572c3beb1e9f52b674d8636533f	tree | snapshot
parent	35141544e2994f0f3b87be7d7c9a43ea3cd9840a	commit | diff

Add ARMv8 Montgomery multiplication module.

Reviewed-by: Rich Salz <rsalz@openssl.org>

Point multiplication

Algorithm 3.36 Window NAF method for point multiplication

INPUT: Window width w, positive integer k, $P \in E(\mathbb{F}_q)$.
OUTPUT: kP.

1. Use Algorithm 3.35 to compute $\mathrm{NAF}_w(k) = \sum_{i=0}^{l-1} k_i 2^i$,
2. Compute $P_i = iP$ for $i \in \{1, 3, 5, \ldots, 2^{w-1} - 1\}$.
3. $Q \leftarrow \infty$.
4. For i from $l-1$ downto 0 do
   4.1 $Q \leftarrow 2Q$.
   4.2 If $k_i \neq 0$ then:
   If $k_i > 0$ then $Q \leftarrow Q + P_{k_i}$;
   Else $Q \leftarrow Q - P_{-k_i}$.
5. Return(Q).

Affine coordinates

OpenSSL implements curve operations as written in P1363.

Addition

Let $P = (x_1, y_1)$, $Q = (x_2, y_2)$ such that $P \neq \pm Q$. Then $P + Q = (x_3, y_3)$ is given by

$$x_3 = \lambda^2 + \lambda + x_1 + x_2 + a_2$$
$$y_3 = \lambda(x_1 + x_3) + x_3 + y_1$$
$$\lambda = \frac{y_1 + y_2}{x_1 + x_2}$$

Doubling

Let $P = (x_1, y_1)$ then $2P = (x_3, y_3)$, where

$$x_3 = \lambda^2 + \lambda + a_2$$
$$y_3 = \lambda(x_1 + x_3) + x_3 + y_1$$
$$\lambda = x_1 + \frac{y_1}{x_1}$$

Lambda coordinates

Affine (Knudsen 1999)

Short affine point $P = (x, y)$ is (x, λ) where $\lambda = x + y/x$.

Projective (Oliveira et al. 2014)

With projective equation

$$(L^2 + LZ + a_2 Z^2)X^2 = X^4 + a_6 Z^4.$$

the λ-projective point $(X_1 : L_1 : Z_1)$ corresponds to the λ-affine point $(X_1/Z_1, L_1/Z_1)$.
The inverse of $(X_1 : L_1 : Z_1)$ is $(X_1 : L_1 + Z_1 : Z_1)$.

Computational costs

Computational costs of elliptic curve operations in various coordinate systems w.r.t. finite field inversions (I), multiplications (M), and squarings (S).

Coordinates	double	add	negate
affine	$1I + 2M + 1S$	$1I + 2M + 1S$	–
LD-projective (mixed)	$4M + 5S$	$8M + 5S$	$1M$
λ-projective (mixed)	$4M + 4S$	$8M + 2S$	–
λ-projective	$4M + 4S$	$11M + 2S$	–

ECC in OpenSSL

```
struct ec_method_st {
...
    int (*point_set_affine_coordinates) (const EC_GROUP *, EC_POINT *,
                                         const BIGNUM *x, const BIGNUM *y,
                                         BN_CTX *);
    int (*point_get_affine_coordinates) (const EC_GROUP *, const EC_POINT *,
                                         BIGNUM *x, BIGNUM *y, BN_CTX *);
...
    int (*add) (const EC_GROUP *, EC_POINT *r, const EC_POINT *a,
                const EC_POINT *b, BN_CTX *);
    int (*dbl) (const EC_GROUP *, EC_POINT *r, const EC_POINT *a, BN_CTX *);
    int (*invert) (const EC_GROUP *, EC_POINT *, BN_CTX *);
...
    int (*is_on_curve) (const EC_GROUP *, const EC_POINT *, BN_CTX *);
...
    int (*make_affine) (const EC_GROUP *, EC_POINT *, BN_CTX *);
    int (*points_make_affine) (const EC_GROUP *, size_t num, EC_POINT *[],
                               BN_CTX *);
...
    int (*mul) (const EC_GROUP *group, EC_POINT *r, const BIGNUM *scalar,
                size_t num, const EC_POINT *points[], const BIGNUM *scalars[],
                BN_CTX *);
    int (*precompute_mult) (EC_GROUP *group, BN_CTX *);
    int (*have_precompute_mult) (const EC_GROUP *group);
    int (*field_mul) (const EC_GROUP *, BIGNUM *r, const BIGNUM *a,
                      const BIGNUM *b, BN_CTX *);
    int (*field_sqr) (const EC_GROUP *, BIGNUM *r, const BIGNUM *a, BN_CTX *);
    int (*field_div) (const EC_GROUP *, BIGNUM *r, const BIGNUM *a,
                      const BIGNUM *b, BN_CTX *);
...
} /* EC_METHOD */ ;
```

Scalar multiplication in OpenSSL

```
/** Computes r = generator * n sum_{i=0}^{num-1} p[i] * m[i]
 *  \param  group  underlying EC_GROUP object
 *  \param  r      EC_POINT object for the result
 *  \param  n      BIGNUM with the multiplier for the group generator (optional)
 *  \param  num    number futher summands
 *  \param  p      array of size num of EC_POINT objects
 *  \param  m      array of size num of BIGNUM objects
 *  \param  ctx    BN_CTX object (optional)
 *  \return 1 on success and 0 if an error occurred
 */
int EC_POINTs_mul(const EC_GROUP *group, EC_POINT *r, const BIGNUM *n,
                  size_t num, const EC_POINT *p[], const BIGNUM *m[],
                  BN_CTX *ctx);
```

Montgomery's Ladder in OpenSSL

```
/*-
 * Computes scalar*point and stores the result in r.
 * point can not equal r.
 * Uses a modified algorithm 2P of
 *     Lopez, J. and Dahab, R.  "Fast multiplication on elliptic curves over
 *     GF(2^m) without precomputation" (CHES '99, LNCS 1717).
 *
 * To protect against side-channel attack the function uses constant time swap,
 * avoiding conditional branches.
 */
static int ec_GF2m_montgomery_point_multiply(const EC_GROUP *group,
                                             EC_POINT *r,
                                             const BIGNUM *scalar,
                                             const EC_POINT *point,
                                             BN_CTX *ctx)
```

Bug attacks and projective randomization

- Bug attacks (Biham et al. 2008) target implementation errors to steal keys.
- Pick β at random, and at the beginning of scalar multiplication set the accumulator to $(\beta X : \beta L : \beta Z)$.
- Observe
  $(\beta X : \beta L : \beta Z) \mapsto ((\beta X)/(\beta Z), (\beta L)/(\beta Z)) = (X/Z, L/Z)$.

ECDH performance

ECDH operations per second. Intel Celeron 2955U 1.40GHz.

curve	stock	modified	gain
nistk163	2107.7	2022.6	-4.0%
nistk233	1675.2	1670.2	-0.3%
nistk283	929.3	921.0	-0.9%
nistk409	589.5	563.8	-4.4%
nistk571	248.7	244.9	-1.5%
nistb163	2043.9	2011.4	-1.6%
nistb233	1600.9	1640.6	2.5%
nistb283	891.6	903.9	1.4%
nistb409	551.9	559.4	1.4%
nistb571	229.1	243.5	6.3%

ECDSA performance

ECDSA operations per second. Intel Celeron 2955U 1.40GHz.

curve	stock (sign)	modified (sign)	gain (sign)	stock (verify)	modified (verify)	gain (verify)
nistk163	2304.1	6723.4	191.8%	1022.9	1617.6	58.1%
nistk233	1146.2	5147.5	349.1%	791.8	1313.5	65.9%
nistk283	770.6	3136.7	307.0%	442.6	744.2	68.1%
nistk409	341.0	1969.2	477.5%	280.2	456.4	62.9%
nistk571	158.2	896.0	466.4%	120.2	199.0	65.6%
nistb163	2300.3	6684.2	190.6%	983.1	1635.9	66.4%
nistb233	1174.2	5227.7	345.2%	765.0	1280.2	67.3%
nistb283	771.3	3142.4	307.4%	420.1	735.1	75.0%
nistb409	339.8	1952.7	474.7%	262.4	446.5	70.2%
nistb571	157.6	858.8	444.9%	111.1	197.7	77.9%

Conclusion

- Source code patches: RT 4013
  `http://marc.info/?l=openssl-dev&m=144008703808363`
- Leverages existing arch, not a(nother) full stack
- Crypto in a vacuum has dubious utility

Future work

Specialized finite field arithmetic (Bluhm & Gueron 2015)